Exercice given at the end of Lecture 1 for Lecture 2.

Let $\mu = (\mu(i); i \geq 0)$ be a probability distribution on $\mathbb{Z}_+$. Denote by $(Z_n)_{n\geq 0}$ the Galton–Watson process with offspring distribution with $Z_0 = 1$ (started with one individual).

(i) Calculate $\mathbb{E}[Z_n]$.

(ii) Calculate $\mathbb{E}[Z_n^2]$.