Week 15: Review exercises

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1 Some review exercises

The solutions of the exercises which have not been solved in some group will be available on the course webpage.

Exercise 1. Complete the boxes $\square$ with $\Longrightarrow$, $\Longleftarrow$, or $\Longleftrightarrow$ and justify your answers:

a) Let $f : E \to F$ be a function. Then $\forall x, y \in E,\ x = y \ \square\ f(x) = f(y)$

b) Let $f : E \to F$ be a one-to-one function. Then $\forall x, y \in E,\ x = y \ \square\ f(x) = f(y)$

c) Let $f : E \to F$ be an onto function. Then $\forall x, y \in E,\ x = y \ \square\ f(x) = f(y)$

d) Let $f : E \to F$ be a bijective function. Then $\forall x, y \in E,\ x = y \ \square\ f(x) = f(y)$

e) Let $f, g : E \to F$ be functions. Then $f = g \ \square\ f(E) = g(E)$.

Exercise 2. For $n \geq 1$, consider the permutation $\begin{array}{ccccc} \sigma & : & \{1,2,\dots,n\} & \to & \{1,2,\dots,n\} \\ & & i & \mapsto & n+1-i \end{array}$

1) Write the cycle decomposition of σ.

2) What is the value of $\varepsilon(\sigma)$?

Exercise 3. Six people each throw a fair dice.

a) What is the probability that there are exactly three sixes?

b) What is the probability that the largest number that appears is at least 4?

c) What is the probability that the largest number that appears is exactly 3?

Exercise 4. What is the coefficient of x^8 in the expansion of $(1 + x^2 + x^3)^{40}$ (you can leave binomial coefficients)? Justify your answer.

Exercise 5. Let $f : E \to F$ be a function. Show that f is one-to-one if and only if $\forall A \subseteq E, A = f^{-1}(f(A))$.