

Addendum to lecture 1

It was claimed that $\phi_n(0)$ converges to a fixed point of ϕ .
Here is a justification:

- We have $\phi_n(0) = \mathbb{P}(Z_n = 0)$. Since $Z_n = 0 \Rightarrow Z_{n+1} = 0$, $\phi_n(0)$ is increasing in n . Since $\phi_n(0) \leq 1$, $\phi_n(0)$ converges to a limit denoted by l .
- We have seen that $\phi_{n+1}(s) = \phi_n(\phi(s)) \quad \forall s \in [0, 1]$.

$$\text{Hence } \phi_{n+1}(s) = \underbrace{\phi \circ \phi \dots \circ \phi}_{n \text{ times}}(s) = \phi(\phi_n(s)).$$

$$\text{Hence } \phi_{n+1}(0) = \phi(\phi_n(0))$$

Since $\phi_n(0) \xrightarrow{n \rightarrow \infty} l$, $\phi_{n+1}(0) \xrightarrow{n \rightarrow \infty} l$ and ϕ is continuous at l , we get $l = \phi(l)$ \square